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RESEARCH MEMORANDUM

INVESTIGATION OF DYNAMIC CHARACTERISTICS OF A

TURBINE-PROPELLER ENGINE

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INTRODUCTION

The dynamics of the engine have become of prime importance in the study of control systems for turbine-propeller engines (reference 1). This importance arises from the fact that a control must be matched to an engine for all conditions and therefore, in order to build good controls, the dynamics of the engine must be known. The time response of the engine speed to a step change in blade angle with fuel flow held constant or in fuel flow with blade angle held constant, and the magnitude of change in engine speed produced by a given change in blade angle or fuel flow (ratio of change in engine speed to change in blade angle or fuel flow) represent two of the most important dynamic characteristics of the engine, time constant and gain factor, respectively.

The results of a previous investigation of a turbine-propeller engine (reference 2) show that values of the time constant can be determined from frequency response analysis of the engine.

The purpose of this investigation was to determine the time constants and gain factors of an engine over an extended range of operating conditions in order to study the dynamics of turbine-propeller engines. In addition, a comparison is made between time constants obtained from steady-state data and time constants obtained by transient operation of the system. The effects of magnitude and direction of change in the input variable on the time constant are discussed.

All turbine-propeller engine characteristics presented in this report were determined under sea-level static conditions.

ANALYSIS

Time Constant from Steady-State Consideration

The first method of obtaining the time constant involves the use of steady-state characteristic curves of the engine-propeller system (fig. 1). These curves show the variation of engine torque with engine rotative speed for values of constant fuel flow and for values of constant blade angle.

An analytical expression relating time constant to the steady-state characteristic curves of the engine system is desired. The first-order linear differential equation representing the dynamic response of the turbine-propeller engine (reference 2) is

$$\frac{\dfrac{I_p}{R^2} + I_e}{\dfrac{1}{R^2}\left(\dfrac{\partial Q_p}{\partial N_p}\right)_\beta - \left(\dfrac{\partial Q_e}{\partial N_e}\right)_{W_f}} \frac{d}{dt} \Delta N_e + \Delta N_e + \frac{\dfrac{1}{R}\left|\dfrac{\partial Q_p}{\partial \beta}\right|_{N_p} \Delta \beta - \left|\dfrac{\partial Q_e}{\partial W_f}\right|_{N_e} \Delta W_f}{\dfrac{1}{R}\left|\dfrac{\partial Q_p}{\partial N_e}\right|_\beta - \left|\dfrac{\partial Q_e}{\partial N_e}\right|_{W_f}} = 0$$

(1)

All symbols are defined in the appendix.

The time constant that characterizes the response of equation (1) therefore is

$$\tau = \frac{\dfrac{I_p}{R^2} + I_e}{\left|\left(\dfrac{\partial Q_p}{\partial N_p}\right)_\beta\right|\dfrac{1}{R^2} + \left|\left(\dfrac{\partial Q_e}{\partial N_e}\right)_{W_f}\right|} = \frac{\dfrac{I_p}{R^2} + I_e}{\left|\left(\dfrac{\partial Q_e}{\partial N_e}\right)_\beta\right| + \left|\left(\dfrac{\partial Q_e}{\partial N_e}\right)_{W_f}\right|}$$

(2)

From known moments of inertia and values of $\dfrac{\partial Q_e}{\partial N_e}$ from the slopes of steady-state characteristic curves, this expression is used to obtain time constants.

Time Constant from Transient Operation

The second method of determining the time constant requires observation of the engine speed response under actual transient conditions of operation. A step input in fuel flow or blade angle results in a speed response that is observed by the use of a photorecording oscillograph.

Although time constants could be most readily determined from measuring the time for the speed change to reach 63.2 percent of its final value, this method is subject to inaccuracy. Therefore, a semilog method developed by Harold Gold of the Lewis laboratory is used. The basis for this method is as follows: A step change in fuel flow or blade angle in an ideal first-order linear system produces an exponential change in speed with time. Such an exponential curve, rising from an initial value at zero time to some final value, is expressed in the following form:

$$N(t) = N_f \left(1 - e^{-\frac{t}{\tau}}\right) + N_i \, e^{-\frac{t}{\tau}}$$

$$\frac{t}{\tau} = \log(N_f - N_i) - \log(N_f - N(t))$$

$$t = \tau \log \frac{(N_f - N_i)}{(N_f - N(t))}$$

(3)

The last form of equation (3) shows that the relation between t and the logarithm of the ratio $\dfrac{N_f - N_i}{N_f - N(t)}$ is that of a straight line where τ is the slope. Figure 2 shows a semilogarithmic plot of typical data, wherein $(N_f - N(t))$ is plotted on the logarithmic scale and time is plotted on the rectangular scale. The locus of $\log(N_f - N(t))$ is observed to be a straight line. Thus the aforementioned method for obtaining the time constant is valid.

In figure 2:

t_1 represents time when $N_f - N(t) = 1$; $t_1 = \tau(\log N_f - \log 1)$

t_2 represents time when $N_f - N(t) = u$; $t_2 = \tau(\log N_f - \log u)$

$t_2 - t_1 = -\tau \log u + \tau \log 1 = \tau(-\log u)$

$t_2 - t_1 = \tau$ when $u = e^{-1} = 0.368$

Therefore the length of the projected segment on the time axis between the points $N_f - N(t) = 1$ and $N_f - N(t) = 0.368$ is the time constant τ. The term $(N_f - N_i)$ is a constant and does not affect the slope τ.

APPARATUS AND PROCEDURE

Engine Installation

The principal components of the engine used in this investigation are an axial-flow compressor, reverse-flow combustion chambers, and a single-stage direct-coupled turbine. A two-stage planetary gear system provided a speed reduction between the turbine and the propeller.

A 12-foot, 1-inch-diameter, four-bladed propeller was installed on the engine. The pitch-changing mechanism maintains a linear relation between the position of the input lever, or beta arm, and the blade angle, as noted in reference 2.

In order to vary blade angle, a hydraulic actuator was connected to the beta arm. Fuel flow was varied by means of a hydraulic actuator attached to the engine fuel regulator valve.

Instrumentation

Steady-state measurements were taken of fuel flow, blade angle, turbine outlet temperature, engine torque, and engine speed. These variables also were measured during transient conditions of operation.

Transient fuel flow was indicated by the use of an aneroid-type pressure sensor with a strain-gage element to measure total fuel pressure in the main fuel line immediately upstream of the engine fuel manifold. The strain-gage element was connected to a bridge circuit in a strain analyzer, where the resulting signal was amplified and fed to a photorecording oscillograph. Steady-state measurement of fuel flow was obtained by use of a rotameter installed in the main fuel line.

Blade-angle position was measured by use of a potentiometer, actuated by the propeller blade, that varied the flow of current in an electric circuit in direct proportion to the blade position. For steady-state measurements, the current was measured by a milliammeter; during transient conditions the circuit was switched to a recording oscillograph element. A conventional slip-ring arrangement was used to complete the circuit between the potentiometer on the propeller hub and the recording device.

The ring gear of the planetary reduction unit of the engine is restrained by a self-balancing hydraulic system. Under transient conditions of operation, the torque output of the engine was indicated by the pressure required to act on the hydraulic piston to maintain balance. This pressure was measured by an aneroid-type pressure sensor with a strain-gage element. The signal was amplified and fed to an oscillograph unit. For steady-stage torque output measurement, the pressure was measured by a Bourdon-type gage. Because this measurement was made under steady-state conditions, it was also an indication of torque input. The methods of analysis applied to this investigation did not require utilization of transient torque measurement.

Transient speed measurement was facilitated by a direct-current generator geared to the engine. The voltage generated was proportional to engine speed. This voltage was applied to an oscillograph element.

Engine speed had a tendency to drift slightly under steady-state conditions, even though input variables were held constant. In order to obtain an accurate indication of steady-state speed immediately

before and after each transient run, a means of effectively increasing
the scale sensitivity of the speed indicator was incorporated in the
measuring system. A mechanical differential and a synchronous electric
motor were utilized in the measuring circuit to make possible speed
measurement from an initial value of 11,000 rpm to a final value of
13,000 rpm, thereby expanding the cruising speed range of the engine
over the full scale range of the indicator. One input shaft of the
differential was driven by an engine tachometer pad; another was driven
by the synchronous motor. By means of proper gear ratios, operation of
the synchronous motor caused 0 rpm of the output shaft of the differ-
ential at 10,500 rpm engine speed. As the engine speed increased, the
differential output shaft imparted rotational motion to a three-phase
tachometer generator, connected to the indicator. Speeds from 11,000
to 13,000 rpm were accurately indicated.

Turbine outlet temperature was measured with one chromel-alumel
thermocouple and recorder for steady-state operation and with three
chromel-constantan thermocouples and oscillograph for transient opera-
tion. All thermocouples were located in the same vertical plane immed-
iately behind the turbine. Transient turbine outlet temperatures were
not utilized in this investigation.

A 10-cycle-per-second timing signal generated by an audio-
oscillator was recorded on the oscillograph film to show the time
variation of the recorded variables.

Steady-state values were recorded before and after each transient
run to provide calibrations for measurement of parameters under tran-
sient conditions.

Table I indicates the steady-state and transient characteristics
of the instruments used.

Procedure

Steady-state runs. - In order to obtain engine time constants from
steady-state characteristics, a map of engine torque against engine
speed was utilized. This map (fig. 1) presents engine torque against
engine speed for lines of constant fuel flow and constant blade angle
in the cruising speed range. The fuel flow varied from 700 to
1250 pounds per hour and the blade angle from 15° to 29°. Maximum
torques in the torque-speed curves were limited by an allowable safe
operating turbine outlet temperature of 1265° F.

Transient runs. - In order to obtain time constants and gain
factors under transient conditions of operation, step changes were made
in speed over the cruising speed range. Both increasing and decreasing
incremental changes in speed at a given operating point were recorded

on a photorecording oscillograph. The magnitude of these changes was approximately 400 rpm. To facilitate comparison of time constants from transient operation with time constants from steady-state characteristics approximately the same ranges of constant fuel flow and constant blade angle were used for both transient and steady-state operation. The values of constant fuel flow and constant blade angle used in the transient runs were

Fuel flow (lb/hr): 700, 750, 820, 883, 950, 1000, 1066, and 1126

Blade angle (deg): 15.00, 17.10, 18.90, 21.00, 23.00, 25.00, and 27.03

A reproduction of typical transient data recorded on the oscillograph is presented in figures 3(a) and 3(b).

Magnitude and direction effects. - Effects of magnitude and direction of speed change on the engine time constant over the cruising engine speed range from 11,000 to 13,000 rpm were investigated. The variations in speed to determine these effects were obtained by varying blade angle while maintaining a constant fuel flow. In order to cover the cruising engine speed range, an operating speed of approximately 12,000 rpm was chosen. Using this value of initial speed, step changes in blade angle first were made to produce incremental changes in speed from 200 to 1000 rpm both above and below this starting point. Using approximately 12,000 rpm as the final value, step changes in blade angle then were made to produce incremental changes in speed of 200 to 1000 rpm from initial speeds above and below this approximate final speed point. Time constants were obtained for all of the incremental speed changes.

RESULTS AND DISCUSSION

Experimentally Determined Time Constant from Steady-State

Characteristics of Engine

Values of time constant from steady-state characteristics for various values of constant fuel flow and constant blade angle are presented in figure 4. All values of time constant are corrected to NACA standard sea-level temperature and pressure.

Experimentally Determined Time Constant from Transient

Operation of Engine

The faired curves in figure 5 show time constant variation with speed for various values of constant fuel flow and constant blade angle from transient operation.

Comparison of Time Constants

A comparison of figures 5(a) and 5(b) indicates that the average value of time constant for constant blade angle exceeds the average value of time constant for constant fuel flow by approximately 13 percent for all values of speed in the engine cruising speed range. Although this correlation is satisfactory for controls work, it indicates a lag in the fuel system as compared to blade angle response.

A comparison of figures 4(a) and 5(a) indicates that the average values of time constant for constant fuel flow from steady-state characteristics agree satisfactorily with average values of time constant from transient operation for all values of speed in the engine cruising speed range. Figure 6(a) presents the variation with speed of the time constant from steady-state characteristics and time constant from transient data at the minimum and maximum values of fuel flow.

A comparison of figures 4(b) and 5(b) indicates that for any given value of constant blade angle the time constants from transient data exceed the time constants from steady-state characteristics by approximately 14 percent for all values of engine speed in the cruising speed range. Figure 6(b) presents variation with speed of time constants from steady-state characteristics and time constants from transient data at the minimum and maximum values of blade angle.

Time Constant Variation with Changes in Magnitude

or Direction of Speed Increments

Figure 7 presents the effect on time constant from transient operation of covering the speed range in increasing and decreasing increments of from 200 to 1000 rpm and for a fuel flow of 950 pounds per hour. The time constants resulting from these incremental changes are presented as a function of the final engine speed and are shown by data points. The behavior of average time constants as a function of final speed for incremental changes of 400 rpm over the cruising speed range is indicated by the line for comparison of time constant at final speed for varied incremental speed changes. The results indicate that increasing or decreasing changes in speed up to and including a magnitude of 1000 rpm can be made with no appreciable variation in the value of the time constant within the experimental error. Furthermore, the variation of time constant versus final engine speed is of such

small magnitude as to indicate that incremental changes in speed larger than 1000 rpm can be made with no appreciable variation in the value of the time constant.

Experimentally Determined Gain Factors from Transient Operation

of Engine

In order to measure the speed-to-blade-angle-gain factor and the speed-to-fuel-flow-gain factor, the initial and final values of speed for known changes in blade angle or fuel flow were utilized. Figures 8 and 9 present speed-to-blade-angle-gain factor and speed-to-fuel-flow-gain factor, respectively, versus speed for the incremental changes in speed used to investigate time constants.

Figure 8 indicates that the speed-to-blade-angle-gain factor varies linearly with engine speed and the rate of change in gain factor with speed is approximately the same for all values of constant fuel flow. The gain factor increases very rapidly both with a decrease in engine speed for a given constant fuel flow and with increasing values of constant fuel flow at a given engine speed. This condition indicates that decidedly large values of gain factor exist at the lower portion of the cruising speed range for high values of fuel flow.

Figure 9 indicates that for values of constant blade angle the speed-to-fuel-flow-gain factor increases as the engine speed decreases. This increase is not linear as the gain factor increases more rapidly for a given blade angle with changes in speed as the engine speed is decreased. For a given value of engine speed the gain factor increases as the value of constant blade angle decreases. The combination of low speeds and small blade angle gives maximum gain factors.

SUMMARY OF RESULTS

The following results apply to time constant and gain factor behavior of a turbine-propeller engine in sea-level static operation:

1. For all values of speed in the engine cruising speed range, average values of time constant for constant fuel flow obtained from both steady-state characteristics and from transient operation agreed satisfactorily, whereas the time constants obtained from transient operation exceeded the time constants obtained from steady-state characteristics by approximately 14 percent for any given blade angle.

2. Over the entire cruising range of the engine, the time constants showed only a small variation with final engine speed and were independent of magnitude and direction of change in blade angle for constant fuel flow.

3. For values of constant fuel flow, speed-to-blade-angle gain increased linearly and rapidly with decreasing engine speed. For values of constant engine speed, this gain increased with fuel flow. Consequently high speed-to-blade-angle gain existed at a combination of low speed and high fuel flow.

4. For values of constant blade angle, speed-to-fuel-flow gain increased rapidly with decreasing speed. The rate of increase of this gain with decreasing speed became greater at the lower values of speed. For values of constant speed, this gain increased with decreasing values of blade angle. Consequently, high speed-to-fuel-flow gain existed at a combination of low speed and low blade angle.

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    National Advisory Committee for Aeronautics,
        Cleveland, Ohio.

APPENDIX - SYMBOLS

I_e polar moment of inertia of engine, (lb)(ft)(sec)(rad)(min)/ revolution

I_p polar moment of inertia of propeller, (lb)(ft)(sec)(rad)(min)/ revolution

N_e engine speed, rpm

N_f final engine speed, rpm

N_i initial speed of engine at time 0, rpm

N_p propeller speed, rpm

$N(t)$ speed of engine at time t, rpm

Q_e torque input to engine, lb-ft

Q_p torque output of propeller, lb-ft

R gear ratio of engine speed to propeller speed

t time, sec

t_1 time when t is t_1, sec

t_2 time when t is t_2, sec

u value of $N_f - N(t)$ when t is t_2

W_f fuel flow, lb/hr

β propeller blade angle, deg

Δ incremental change

δ $\dfrac{\text{ambient static pressure}}{\text{NACA standard sea level pressure}}$

θ $\dfrac{\text{ambient static temperature}}{\text{NACA standard sea level temperature}}$

τ system time constant, sec

REFERENCES

1. Lazar, James, and De Rocher, Wilfred L., Jr.: Correlation of Analog Solutions with Experimental Sea-Level Transient Data for Controlled Turbine-Propeller Engine, Including Analog Results at Altitudes. NACA RM E51B08, 1951.

2. Taylor, Burt L., III, and Oppenheimer, Frank L.: Investigation of Frequency-Response Characteristics of Engine Speed for a Typical Turbine-Propeller Engine. NACA TN 2184, 1950.

TABLE I. - STEADY-STATE AND TRANSIENT CHARACTERISTICS OF INSTRUMENTS

Measured quantity	Steady-state instrumentation	Transient instrumentation	
		Sensor	Frequency response range (cycles/sec)
Fuel flow (static pressure)	Rotameter	Aneroid-type pressure sensor with strain-gage element connected to give indication on oscillograph	0 - 40
Blade angle	Wire-wound potentiometer connected to give position indication on microammeter	Wire-wound potentiometer connect to give position indication on oscillograph	0 - 40
Turbine-outlet temperature	One chromel-alumel thermocouple connected to recorder	Three chromel-constantan thermocouples in series connected to give indication on oscillograph	0 - 1
Torque	Bourdon-type gage	Aneroid-type pressure sensor with strain-gage element connected to give indication on oscillograph	0 - 40
Engine speed	Three-phase tachometer generator, mechanical differential and synchronous motor to eliminate sensing of speeds below desired range	Direct-current generator connected to give indication on oscillograph	Limited by filter circiut 0 - 2.65
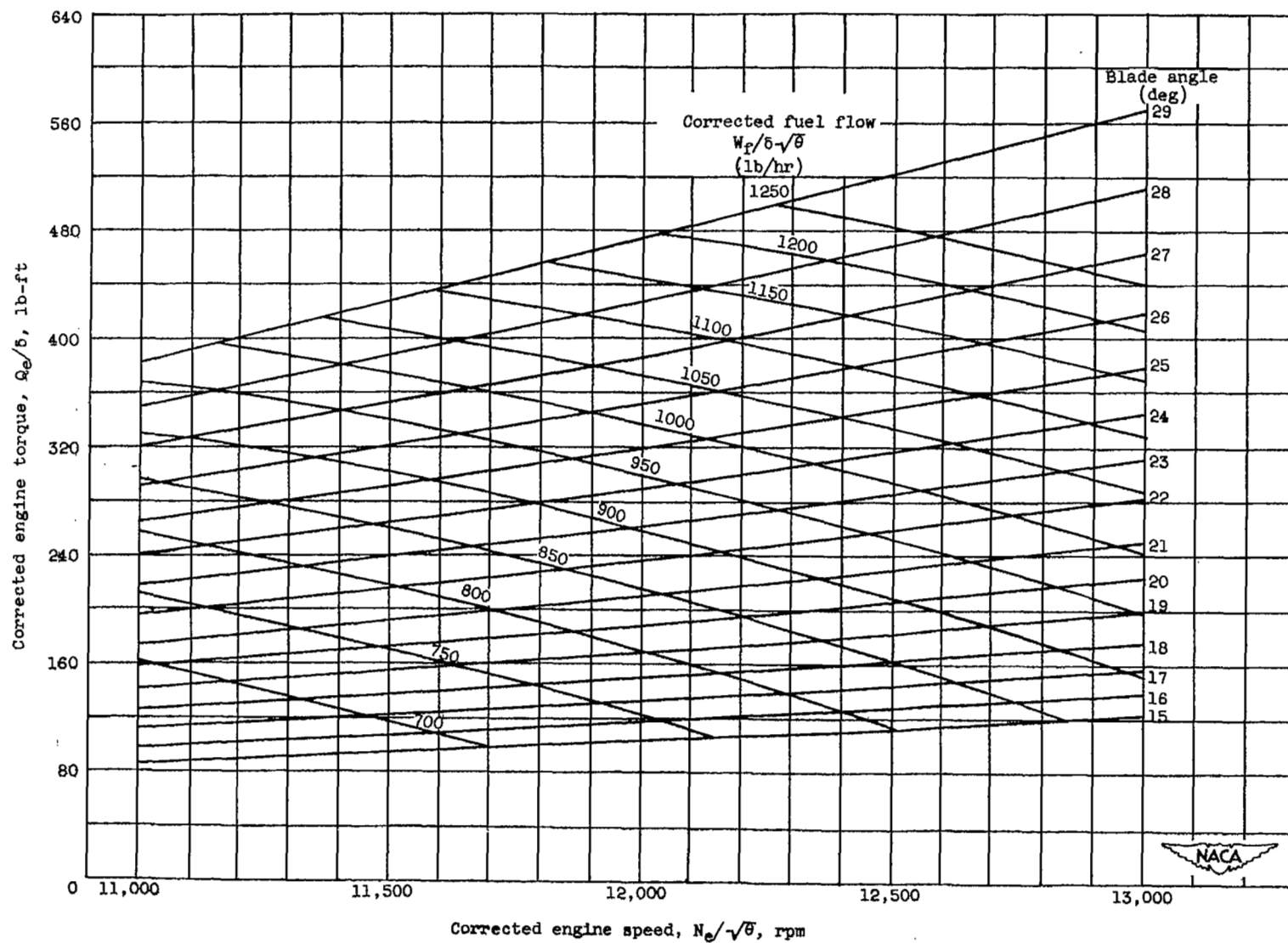
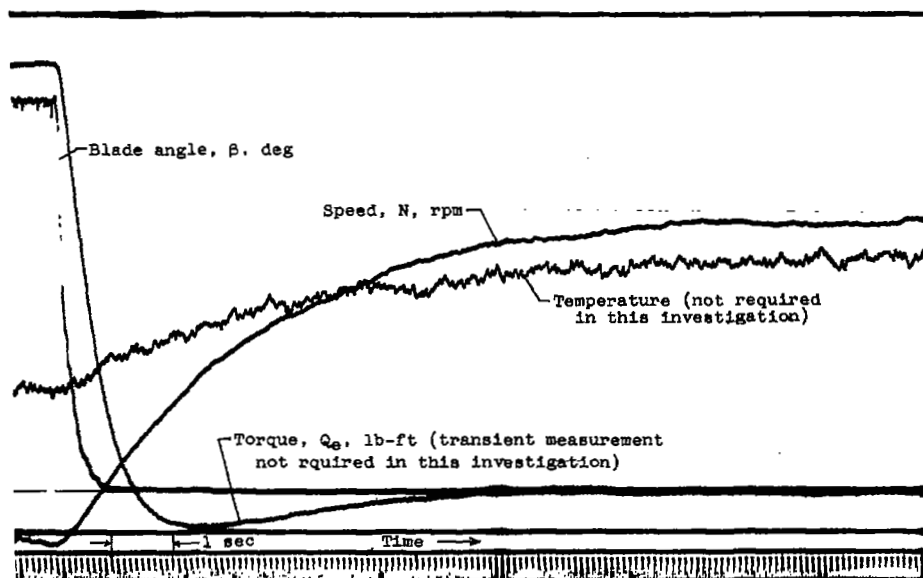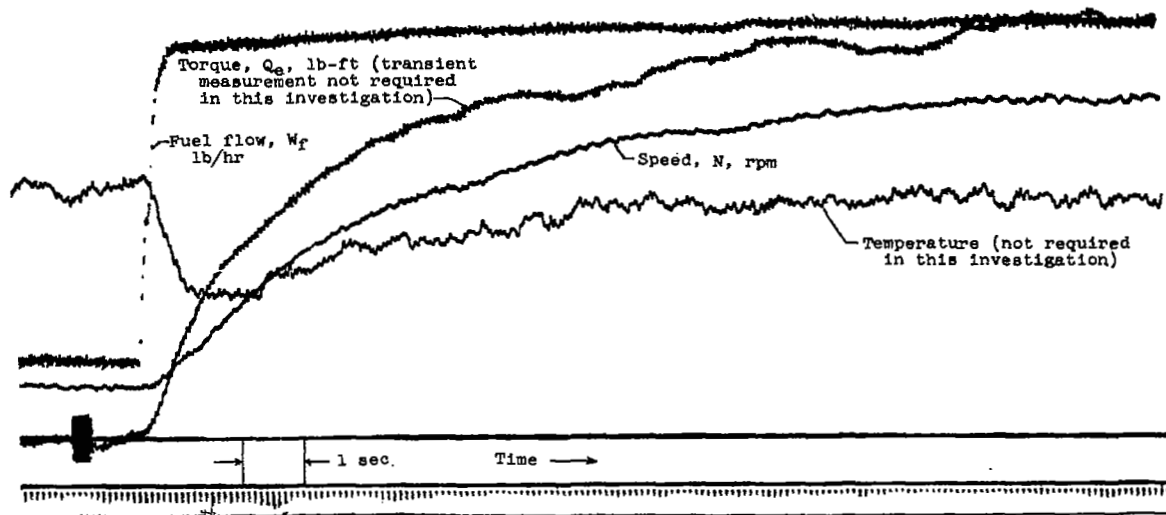Figure 1. - Steady-state torque-speed characteristics of turbine-propeller engine.

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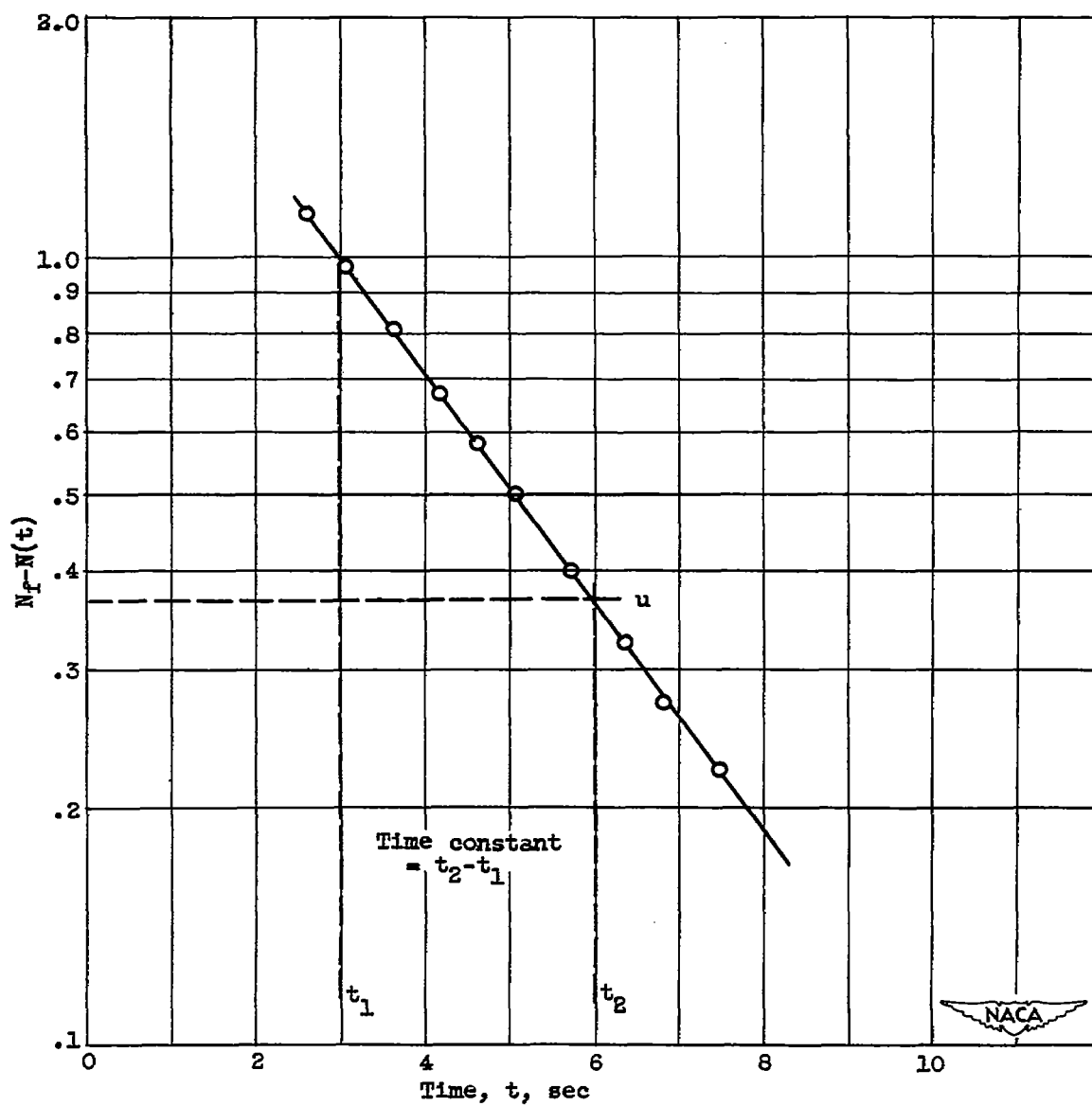
Figure 2. - Semilogarithmic variation of $\left[N_f - N(t)\right]$ with time.

Blade angle, β. deg

Speed, N, rpm

Temperature (not required in this investigation)

Torque, Q_e, lb-ft (transient measurement not rquired in this investigation)

1 sec　　　　Time

(a) Step input in blade angle at constant fuel flow.

Torque, Q_e, lb-ft (transient measurement not required in this investigation)

Fuel flow, W_f lb/hr

Speed, N, rpm

Temperature (not required in this investigation)

1 sec.　　　　Time

(b) Step input in fuel flow at constant blade angle.

Figure 3. - Typical recording of transient data from turbine-propeller engine.

(a) Constant fuel flow.

(b) Constant blade angle.
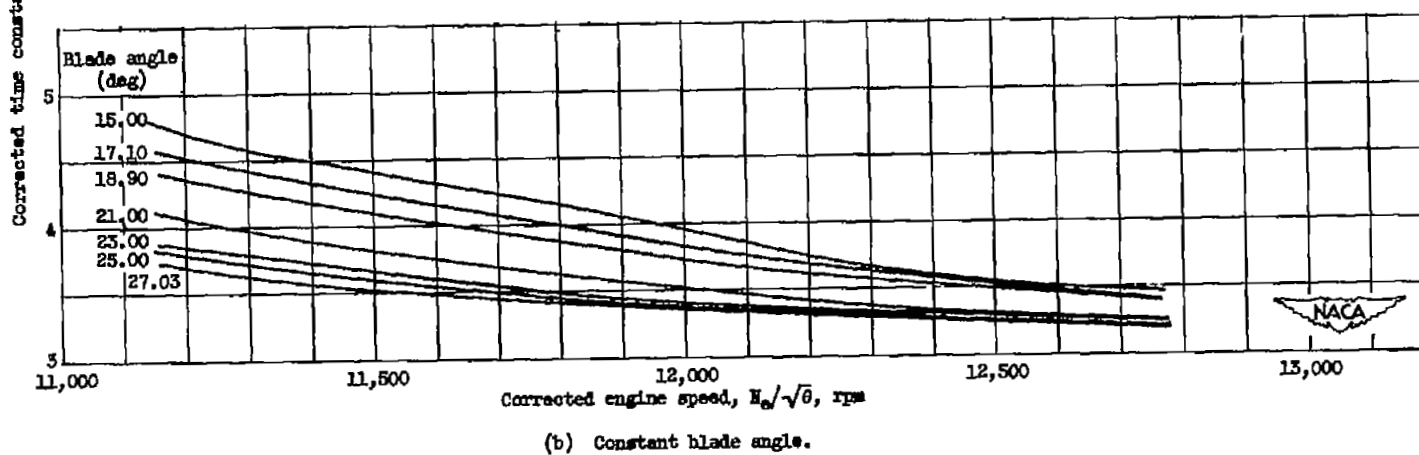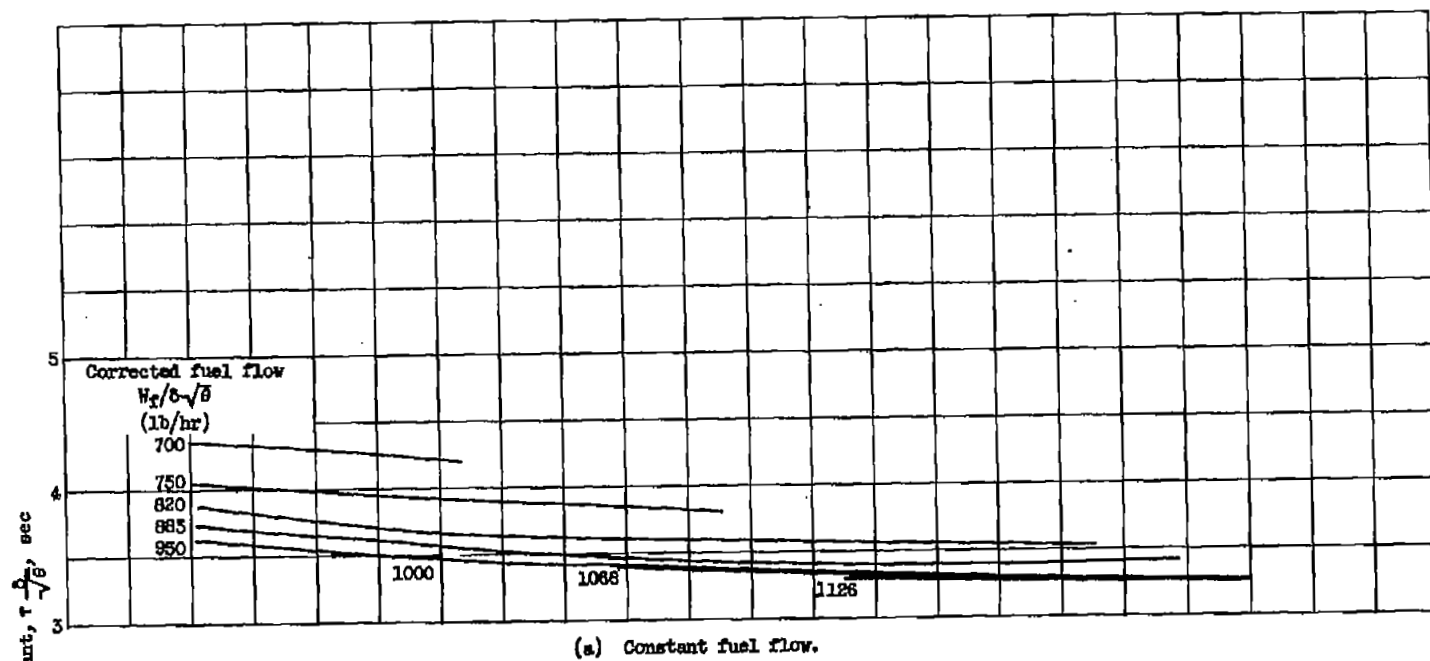
Figure 4. - Variation of time constant with engine speed for turbine-propeller engine from steady-state characteristics.
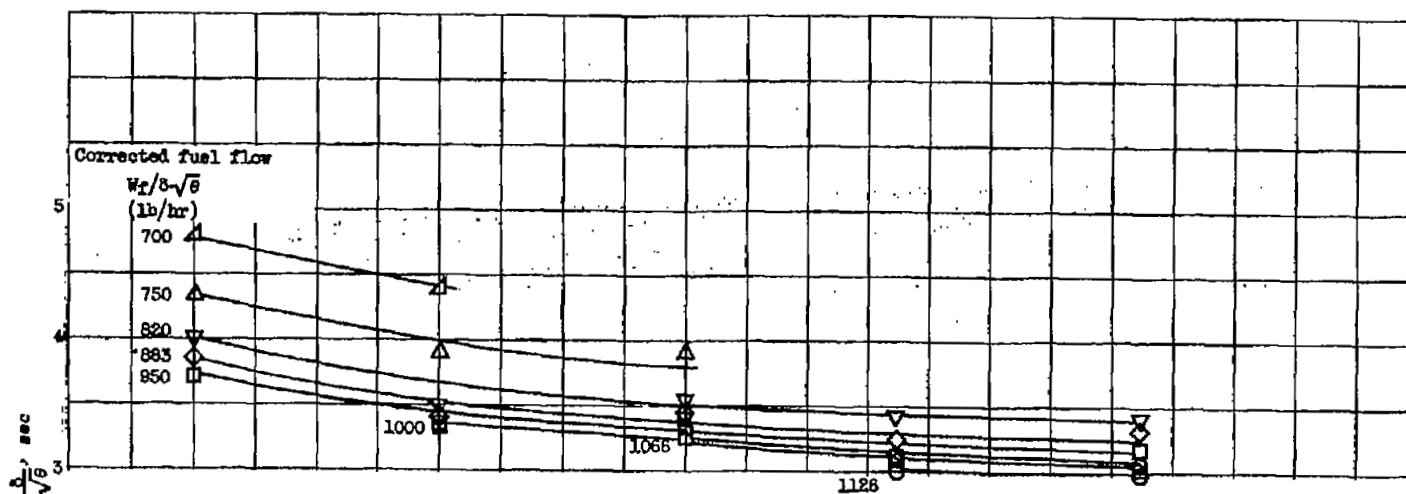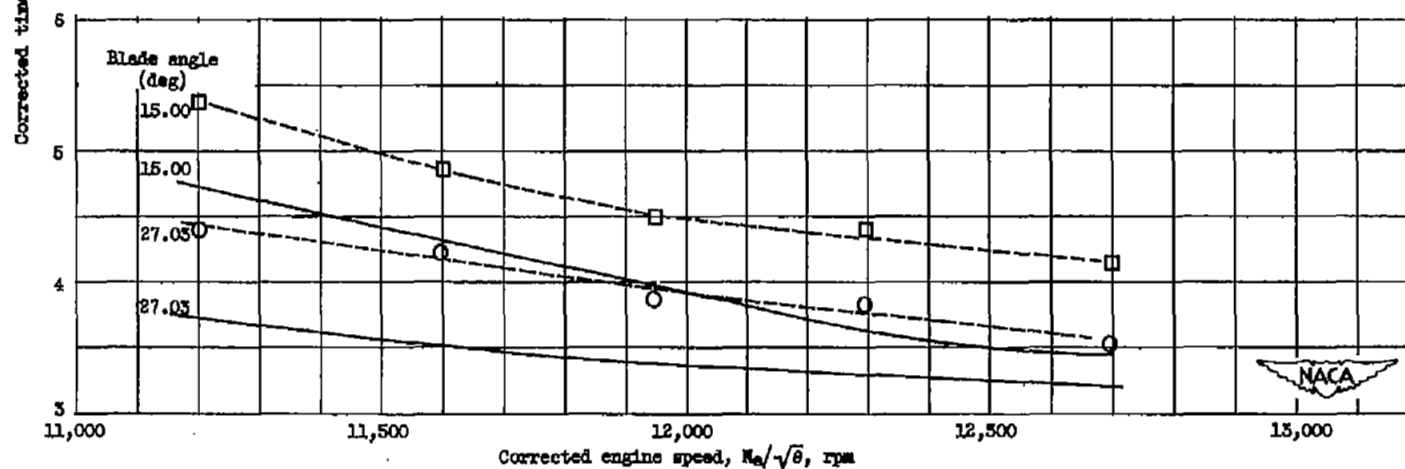
(a) Constant fuel flow.
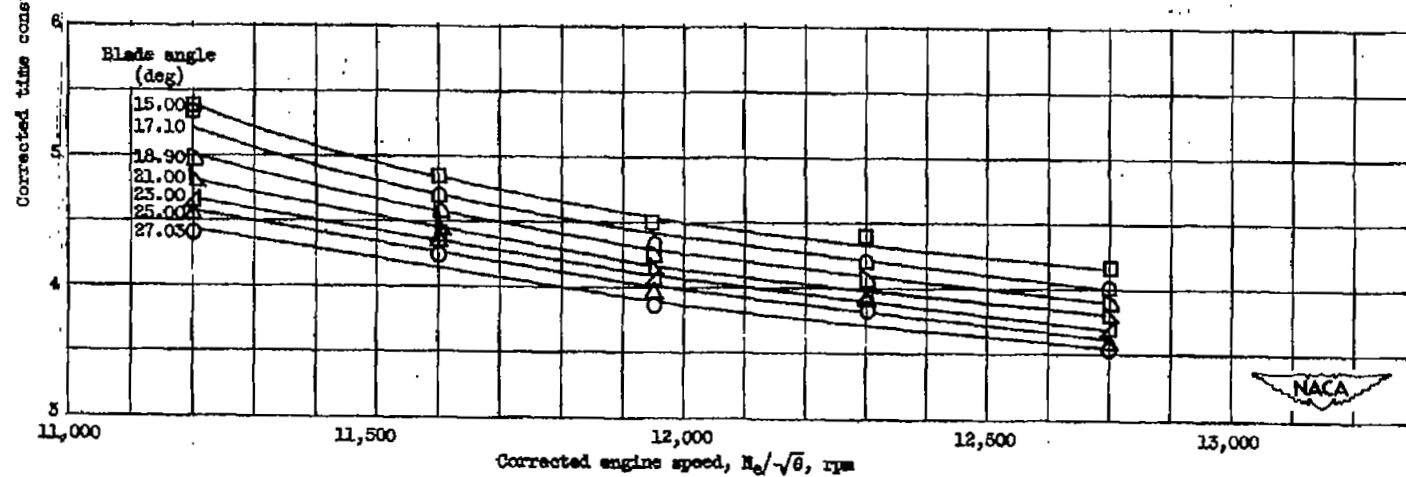
(b) Constant blade angle.

Figure 5. - Variation of time constant with engine speed for turbine-propeller engine from transient operation. Data points shown are averaged values.
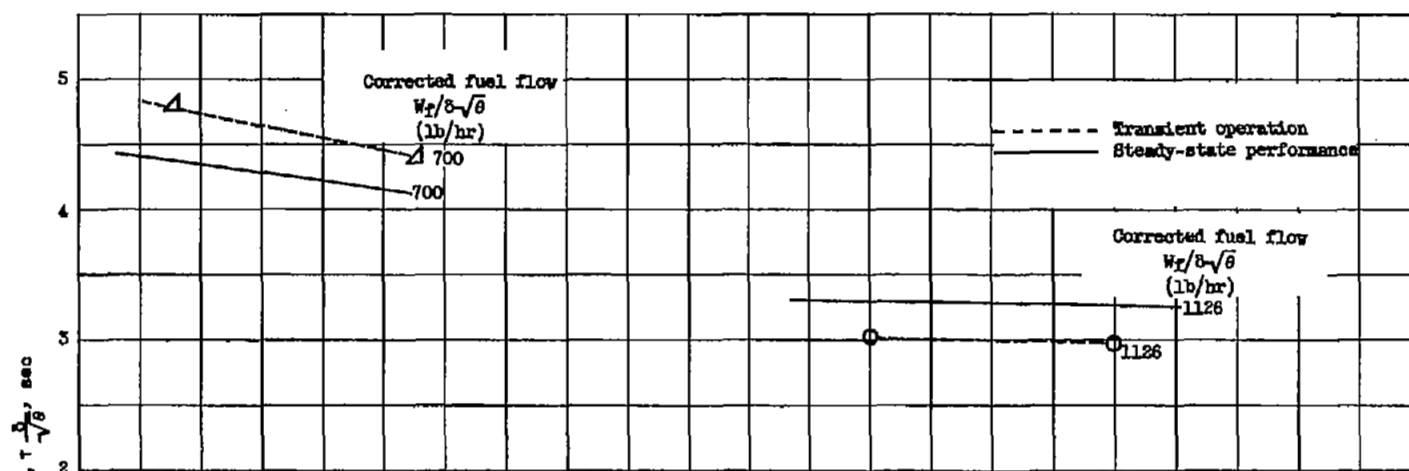
(a) Upper and lower constant fuel flow limits.

(b) Upper and lower constant blade angle limits.

Figure 6. - Variation of time constants from steady-state characteristics and from transient operation with engine speed for turbine-propeller engine. Data points shown are averaged values.
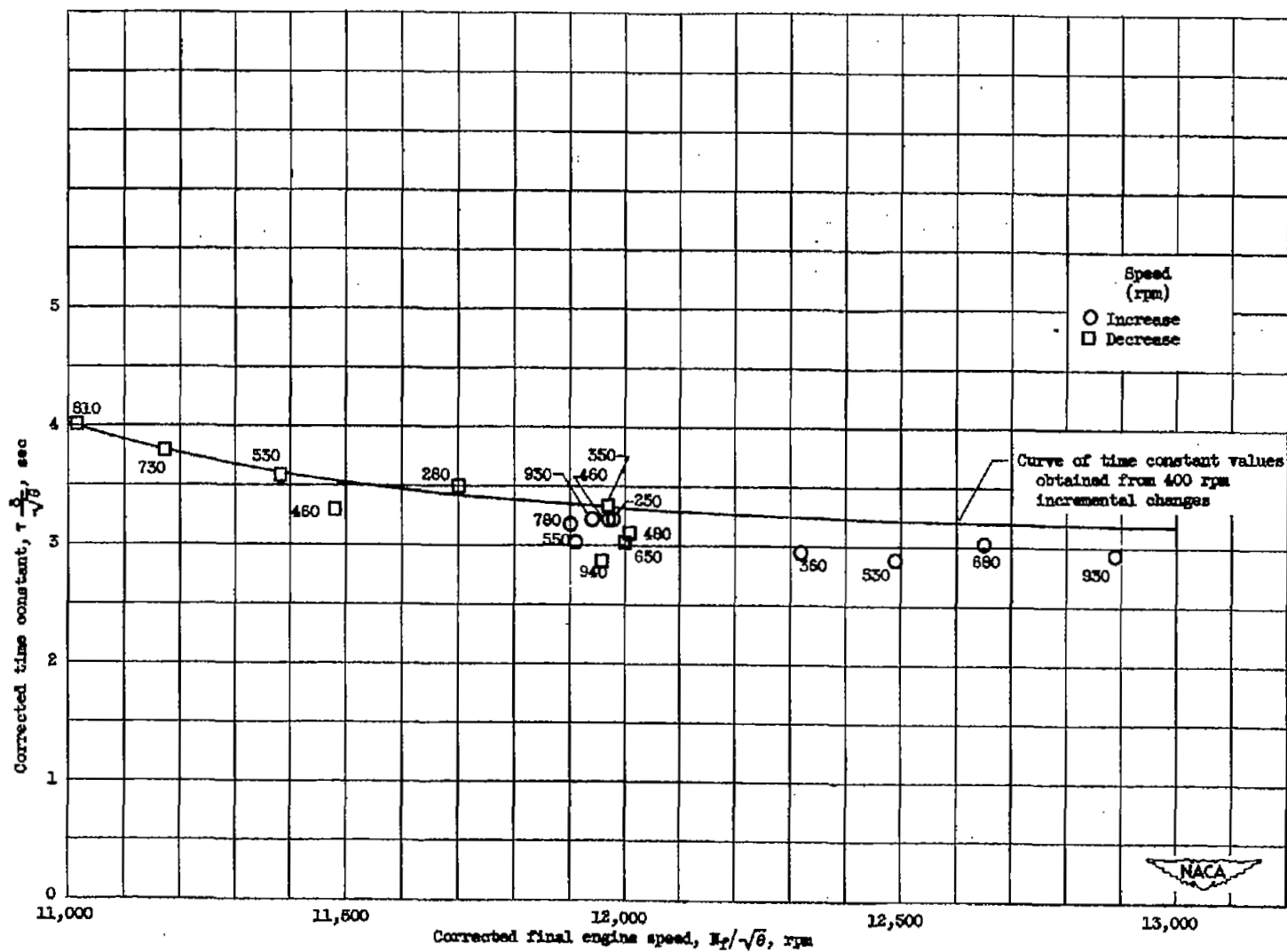
Figure 7. - Variation of time constant with final engine speed for turbine-propeller engine from transient operation of engine with increasing incremental changes in speed at constant fuel flow of 950 pounds per hour.
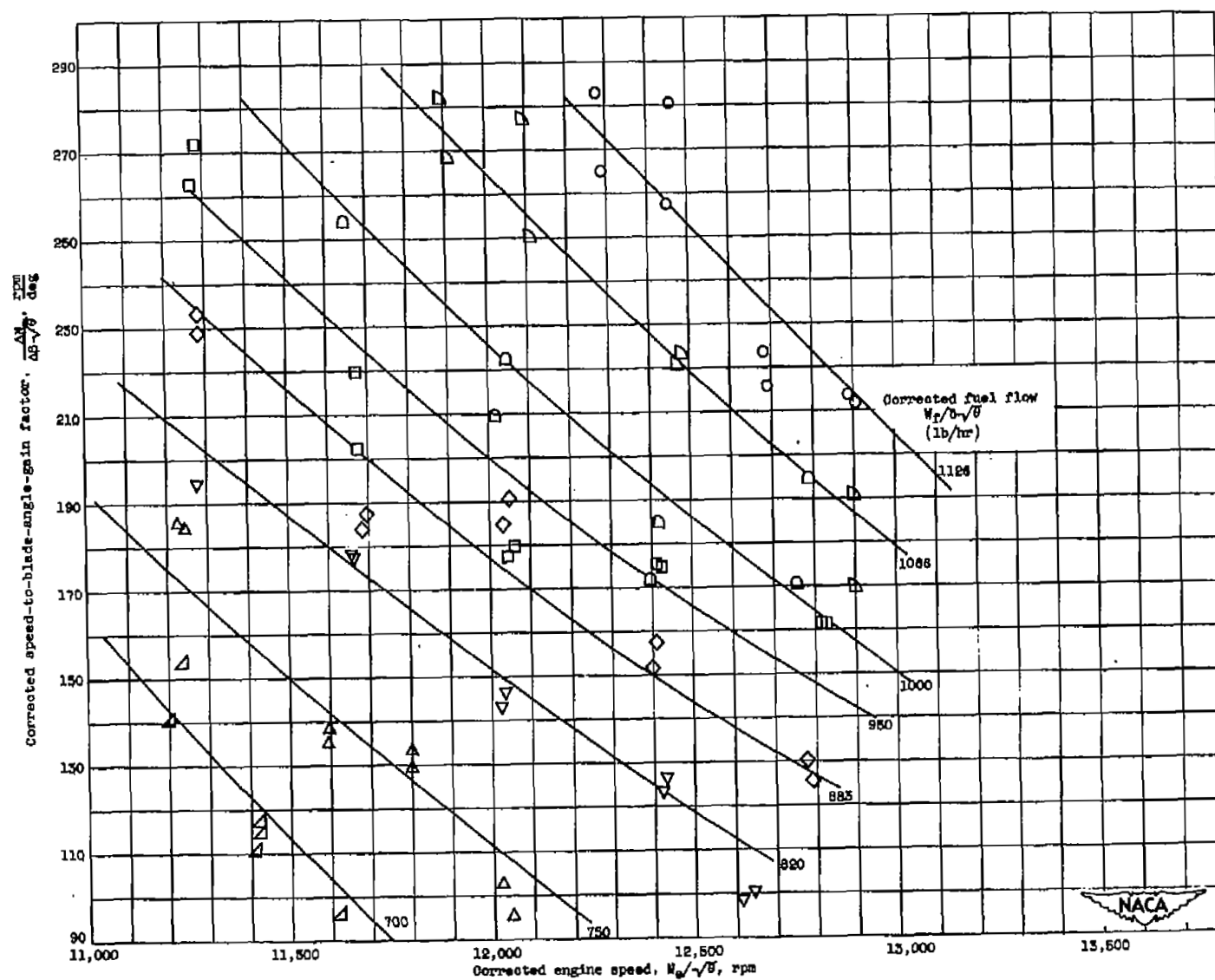
Figure 8. - Variation of speed-to-blade-angle-gain factor with engine speed for turbine-propeller engine from transient operation at constant fuel flow.
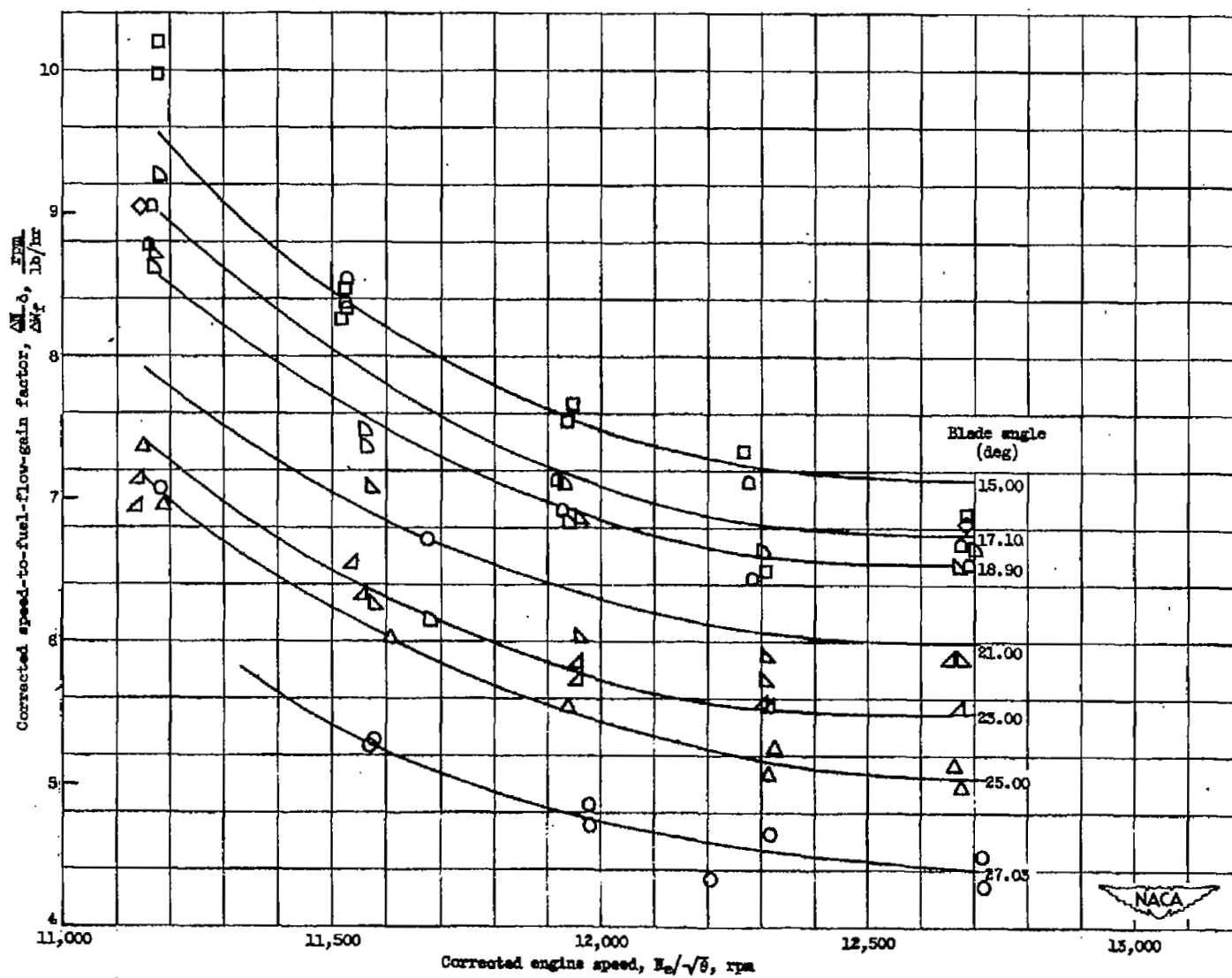
Figure 9. - Variation of speed-to-fuel-flow-gain factor with engine speed for turbine-propeller engine from transient operation at constant blade angle.